Q2.

For each integer n = 1,2,3,…, 7, determine whether there exists a red-black tree having exactly n nodes, with *all of them black.* Fill out the chart below to tabulate the results:

|  |  |
| --- | --- |
| Num Nodes n | If there exists a red-black tree with all nodes black |
| 1 | Yes |
| 2 | No |
| 3 | No |
| 4 | Yes |
| 5 | No |
| 6 | No |
| 7 | No |

Q3. For each integer n = 1,2,3,…, 7, determine whether there exists a red-black tree having exactly n nodes and exactly one red node*.* Fill out the chart below to tabulate the results:

|  |  |
| --- | --- |
| Num Nodes n | If there exists a red-black tree with exactly one red node |
| 1 | No |
| 2 | Yes |
| 3 | No |
| 4 | Yes |
| 5 | Yes |
| 6 | No |
| 7 | Yes |

Q4

Show the red black tree that results after each of the integer keys 21,32,64,75 and 15 inserted in that c into an initially empty red black tree. Clearly show the tree that results after each insertion (indicating the col each node), and make clear any rotations that must be performed

To solve Question 4, we will simulate the step-by-step insertion of the keys into an initially empty red-black tree while ensuring the tree adheres to red-black tree properties:

1. Every node is either red or black.
2. The root is always black.
3. Red nodes cannot have red children (no two consecutive red nodes).
4. Every path from a node to its descendant NULL nodes has the same number of black nodes.
5. When a violation occurs, rotations and/or recoloring are applied.
6. Insert 21, which becomes the root which is always black 21(Black)
7. Insert 32, to the right of 21. New nodes are red by default. No property violations.

21(Black)

32(Red)

1. Insert 64 to the right of 32, new nodes are red by default. It violates the property check because of two consecutive red nodes(32 and 64). Performing a resolution, and rotation: Left rotation at 21, recolor 32 to black.

32(Black)

21(Red) 64(Red)

1. Insert 75, added to the right of 64. New nodes are red. Violation because of two consecutive reds. Recolor 64 and 21 to black. Recolor 32 to red (if necessary, but in this case, the root remains black to maintain balance).

32(Black)

21

(

Black

**)**

64

(

Black

**)**

75(Red)

1. Insert 15, its by default red, and you insert it to the left of 21/

32(Black)

21

(

Black

**)**

64

(

Black

**)**

15(Red) 75(Red)